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”OPTIMAL BALANCE VIA ADIABATIC INVARIANCE OF APPROXIMATE SLOW MANIFOLDS”

Abstract:

We analyze the method of optimal balance which was introduced by Viu´dez and Dritschel (J. Fluid Mech. 521, 2004, pp. 343–352) to provide balanced initializations for two-dimensional and three-dimensional geophysical flows, here in the simpler context of a finite dimensional Hamiltonian two-scale system with strong gyroscopic forces. It is well known that when the potential is analytic, such systems have an approximate slow manifold that is defined up to terms that are exponentially small with respect to the scale separation parameter. The method of optimal balance relies on the observation that the approximate slow manifold remains an adiabatic invariant under slow deformations of the nonlinear interactions. The method is formulated as a boundary value problem for a homotopic deformation of the system from a linear regime, where the slow-fast splitting is known exactly, to the full nonlinear regime. We show that, providing the ramp function which defines the homotopy is of Gevrey class 2 and satisfies vanishing conditions to all orders at the temporal end points, the solution of the optimal balance boundary value problem yields a point on the approximate slow manifold that is exponentially close to the approximation to the slow manifold via exponential asymptotics, albeit with a smaller power of the small parameter in the exponent. In general, the order of accuracy of optimal balance is limited by the order of vanishing derivatives of the ramp function at the temporal end points. We also give a numerical demonstration of the efficacy of optimal balance, showing the dependence of accuracy on the ramp time and the ramp function.